FREQUENCY STANDARDS, TIMEKEEPING, AND TRACEABLE SERVICES AT THE NATIONAL RESEARCH COUNCIL OF CANADA

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Abstract

Canadian frequency metrology services and Canada's official time services are both provided by the National Research Council of Canada (NRC). NRC designs, builds, and operates a group of laboratory frequency standards, both primary cesium clocks and hydrogen masers. It disseminates the results as both time and traceable frequency. NRC has an active research and development program in time and frequency technology: on frequency standards (microwave and optical) and on calibration and dissemination techniques with complete uncertainty budgets. Its present and planned capabilities are presented and discussed.

INTRODUCTION

The National Research Council of Canada provides accurate time and frequency services for Canada, which are also available to interested parties in Mexico and the United States under the terms of the North American Free Trade Agreement. The services are based on the capabilities within NRC's time laboratory, on its capabilities to intercompare and coordinate with other laboratories, and on its ability to deliver those services to clients.

CESIUM CLOCKS

One major difference between PTTI work at NRC compared to many other time laboratories is the presence of three continuously operated laboratory cesium-beam standards designed and built at NRC 20 to 25 years ago. They are large, classical magnetic-dipole cesium clocks, 2 to 5 m in overall length, with provision for beam-reversal (to evaluate the microwave cavity phase shift). The larger clock, CsV, has a Ramsey interaction length of 2.1 meters and a linewidth (FWHM) of 60 Hz[1] and an independent standard uncertainty in SI average frequency of 5×10^{-14} for times longer than 1 day. The two smaller clocks (CsVI-A and CsVI-C) each have an interaction length of 1 meter and a linewidth of 90 Hz[2] with an independent standard uncertainty in SI average frequency of 7×10^{-14} for times longer than 1 day. These standards

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are operated to be dominated by white frequency noise of $3\times10^{-12}\tau^{-1/2}$, which suffices to characterize directly most frequency standards, except hydrogen masers.

The past year has been one of major building reconstruction and CsV has been assaulted by each of the "elements" identified by ancient Greece: air, earth, water, and fire. We have taken this time to build (and rebuild) an environmental chamber for CsV which should control the temperature and temperature gradients. We expect CsV to be capable of a flicker-frequency noise floor well below the 10^{-14} level that we commonly observed when the room temperature was not well controlled.

In addition to these primary cesium clocks, there are experimental systems. CsVI-B was withdrawn from service as a primary clock and dismantled. Some of its parts likely will be recycled in the future in an NRC cesium-fountain frequency standard. CsVII was an optically pumped short (20 cm interaction length) frequency standard which did not meet its optimistic design goals for frequency stability. A magneto-optic trap and optical molasses system has been in operation for 2 years[3] and has developed the ability to trap 10^7 cesium atoms per second, confine them in a 1 mm ball, cool them to less than 5 microkelvin, and re-accelerate them (at 4000 m/s^2) to speeds of 7 m/s–all with an optical power low enough (6 mW per beam) to be easily compatible with cryogenic operation. This work is aimed at a tall cesium fountain (2.5 m overall trajectory height, with 1.2 m above the microwave interaction region to give a 1-second Ramsey interaction time or a 0.5 Hz linewidth).

Our laboratory uses two HP5071A "high performance" commercial Cs clocks, one in the main time laboratory and one in the adjunct laboratory 22 km distant at CHU, the NRC shortwave radio station that continuously broadcasts its time and frequency references.

HYDROGEN MASERS

NRC operates three home-built hydrogen masers. One is 30 years old and is operated largely to allow three-cornered-hat measurements of the short-term stability of the two more modern masers[4], which are operated in the cavity auto-tuning mode as clocks. Their stability at 10 days is generally less than 2×10^{-15}. One (H3) is fitted with a cavity coated with FEP-120 Teflon and the other (H4) is coated with Fluoroplast F-10 with a wall shift about half that of the FEP-120 Teflon. The masers have proved useful in evaluating the time and frequency transfer characteristics of the Geodetic GPS measurement systems, in some optical frequency measurements[5], and evaluating hydrogen masers for clients. They are planned for use in frequency intercomparison from cesium fountains.[6,7,8]

PHASE COMPARATORS

The short-term phase stability of clients' frequency standards can be evaluated at 5 MHz using our multichannel phase comparator. Averaging for 1 s, it is dominated by a white phase noise, having a modified Allan deviation of less than $3\times10^{-12}\tau^{-1.5}$ between any two channels. Redundant computers record the data stream from the phase comparator. Shorter-term and higher-stability intercomparisons (such as for hydrogen masers) are measured separately. Longer-term phase comparisons are routinely made in a fully automated way by counting down to 1 pps and comparing with 1 pps signals from other clocks.

LASER-COOLED TRAPPED SINGLE-ION OPTICAL FREQUENCY WORK

A single, trapped $^{138}Ba^+$ ion was laser-cooled to 15 mK and measurements were made of the linewidth of several Zeeman components and the center frequency of the 5d $^2D_{3/2}$ - 5d $^2D_{5/2}$ clock transition (with a wavelength of 12 m). The best value for the FWHM of a Zeeman component and the center frequency of the transition were 5.8 kHz and 24,012,048,317,170 $\pm$ 440 Hz (1.8×10^{-11}), respectively. The frequency measurements were made by direct comparison to cesium using the NRC frequency chain.

Recently, similar work on the 674 nm 5s $^2S_{1/2}$-4d $2D_{5/2}$ clock transition of an $^{88}Sr^+$ ion has yielded 500 Hz for the FWHM of a Zeeman component. The local oscillator used to interrogate the single ion is a cavity-locked laser which is predictable relative to the Sr^+ clock transition to within 2×10^{-12} over many hours. The ion temperature is near 10 mK. Frequency measurements 444,779,043,984 $\pm$ 28 kHz (6.3×10^{-11}) for the transition center have been made with respect to an iodine stabilized HeNe laser at 474 THz, with absolute measurements relative to Cs planned. The Sr^+ single-ion frequency reference at 445 THz and the link to 474 THz is planned to be developed and maintained as a simplified calibration tool for the iodine-stabilized He-Ne lasers at 474 THz, which are commonly used in length metrology. Part of the chain exploits a 2022 nm laser (whose third harmonic is beat with the laser interrogating the Sr^+ resonance), which will be used to develop frequency standards at other frequencies of interest for telecommunications technology.

COMMON-VIEW GPS

NRC's major international time intercomparison link is through a common-view GPS receiver, operated on the tracking schedule prescribed for Eastern North America by the Bureau International des Poids et Mesures. Some of NRC's clients also use this tracking schedule.

Antenna and receiver failures have necessitated unit substitutions. Whenever possible, relative calibrations of replaced antenna components were checked using a Mitrex modem, upconverting to the GPS L1 frequency. Since the Mitrex chipping rate is 2.5/1.023 times the GPS C/A chipping rate, a simple GPS code simulator was built and tested. With a data modulator (driven from an appropriate 50 bps GPS navigation message), we feel that this should evolve into a useful calibration tool.

GEODETIC GPS FREQUENCY TRANSFER

NRC has been collaborating with the geodesists of Canada's Department of Natural Resources on the frequency-transfer characteristics of their carrier-phase-smoothed techniques. In these tests we have been using 8- and 12-channel, dual-frequency, Turborogue GPS receivers. Both code pseudoranges and the integrated carrier phase are recorded in the "geodetic" mode, where each receiver's 20.456 MHz sampling clock is phase-locked to a 5 MHz frequency reference (at many sites from H masers), and the absolute timing is measured from a 1 pps gated out from the receiver's clock. This mode seems to avoid limitations that have been reported for timing receivers exploiting the same technology. The stability of the timing behavior of some of these receivers has been criticized, and after power-down (and at random other times at some sites) the relative phase of the receiver clock and the reference is lost. In an attempt to avoid the difficulties associated with determining the relative phase of the 20.456 MHz and the

reference 5 MHz (in hard-to-resolve steps of 78.2 ps), we have designed and built an external 20.456 MHz synthesizer which can be phase-reset by an external 1 pps clearing the synthesizer registers. If the 1 pps is phased correctly with respect to the reset, in a 200-ns window, we expect the only phase ambiguities left in a receiver using this system will be 48.885-ns steps, which are much simpler to resolve with the GPS receiver itself.

In this mode, we have performed zero-baseline tests using two Turborogue SNR-8000 receivers sharing a single antenna and separately locked to two outputs of a single maser. The differential noise level and stability are quite satisfactory[9] without any special measures having been taken.

Until recently, full receiver measurements have been recorded every 30 seconds. They are preprocessed into a set of carrier-smoothed pseudorange measurements every 7.5 minutes. The global solution was done using GIPSY software (developed at JPL) in a postprocessing mode. Unlike many other geodetic analyses, the fitting was done independently for each 24-hour period, fitting observations from 24 stations around the world selected from the shared database of the International GPS Service for Geodynamics (IGS). An unconstrained discontinuity is allowed from one day's clock intercomparison solution to the next day's solution. The histogram of discontinuities is best described by identifying the end-of-day rms deviation of the solution as 310 ps (i.e. a end-of-day jump size of 440 ps), with occasional outliers.

The daily global solution has a wide-open timing filter (1 ms white phase noise allowance), and yet the fitted time differences between two maser-equipped receiver stations show excellent post-fit stability. If the day-to-day discontinuities are discounted as being in some sense "fixable," then the Allan deviation of the residuals for the short term (7.5 minutes to 1 day) is $8\times10^{-13}\tau^{-1/2}$ (white FM noise, τ in seconds–$\sigma_y(\tau = 1 \text{ day}) = 2.7\times10^{-15}$), or 10 times more stable than a high-performance-option HP5071A. If all the discontinuities are included for the same data, the Allan deviation of the residuals is about five times worse: $4\times10^{-12}\tau^{-1/2}$ (from 7.5 minutes to 1 day), and $\sigma_y(\tau = 1 \text{ day}) = 1.4\times10^{-14}$. For times beyond one day, even well-maintained masers cannot be relied upon to give a negligible contribution to the measured stability of the receiver clock differences, and so we have not continued our stability analysis beyond 1 day.

Likely, somewhere between these two limits is the frequency stability of this geodetic methodology. The traditional stability analysis, used above, does not cope well with the solution discontinuities. We use caution in interpreting these results, and we are more comfortable with two other statistical measures which exploit the nearly complete independence of the daily fitting procedures (the previous day's prediction only provides the initial estimates of the satellite orbits for the following day's solution).

Each day's solution also gives a 24-hour average frequency difference between any two stations, and rms averages of the first difference of these can be used to give the Allan deviation at 24 hours, $\sigma_y(\tau = 24\text{ h})$, completely rigorously. The Allan deviations are shown for several different baselines in Table 1. It is quite encouraging to find Allan deviations at one day of 5×10^{-15} for baselines of 4,000 and 6,000 km, and of 7×10^{-15} for a 17,000 km (great-circle) baseline, with no common-view satellites.

As part of the Canadian Active Control System, a wide-area differential GPS system under development at the Geodetic Survey for use across Canada, we have started collecting full Turborogue data at 1-second intervals (up to 12 channels of C/A code pseudorange, C/A carrier phase, L1/L2 delay and–if available–the L1 and L2 P-code pseudoranges and integrated carrier phase data...). The objective is a real-time Canada-wide differential GPS overlay. Subsets of the full receiver data (e.g. RINEX) are not sufficient for the preprocessing that is normally done. We plan to use the same 1-second observations for reporting to BIPM with their common-view tracking schedule. We believe that absolute GPS calibration is important, and plan to work on

the more important influence parameters: multipath, filter, and correlator variations.

GEOSYNCHRONOUS SATELLITE TWO-WAY TIME TRANSFER

Two-way time transfer at 14/12 GHz is done routinely with USNO and NIST. Data acquisition was begun in 1989, and has continued three times per week. The data acquisition is largely automated, but the subsequent data analysis is not, and full analysis is not used routinely at NRC. In our plans, it is a technique with promise for the inter-laboratory intercomparisons expected with cesium-fountain frequency standards, but we believe that the currently identified needs of most Canadian clients can likely be met more economically with development of geodetic GPS techniques.

STANDARD INCERTAINTIES FOR TIME AND FREQUENCY METROLOGY

We have developed and used a rigorous analytic method for calculating the noise contribution to the standard uncertainty of a time or average frequency in the presence of non-white noise (white PM + flicker PM + white FM + flicker FM + random walk FM) which can be used for many interpolation or extrapolation procedures that are common in time and frequency applications. The method is easier to use than simulations, and unlike simulations the new method can converge to an extent which allows it to be used for optimizing weights in fitting procedures.[10] We have used this procedure to analyze the standard uncertainty associated with the crystal and maser oscillators which are associated with a pulsed frequency standard that is intermittently operated.[11,6,7,8] We have extended this work to develop a simple procedure for converting stability measures into the standard uncertainty for transferring average frequency between two general time intervals.

OBTAINING STANDARD UNCERTAINTY FROM THE ALLAN DEVIATION

In metrology, the average frequency is often calibrated at one time interval and used at another interval often shorter than the first interval, and perhaps much later than the first. The transfer of average frequency from one interval to another is a process that is closely related to the stability of the frequency standard used to effect the frequency transfer. The Allan deviation or the modified Allan deviation is commonly used to characterize the stability, but there has been no easy way of converting this knowledge into a rigorous estimate of the standard uncertainty in the general case. Clients of average frequency (including almost all metrology) want to have the standard uncertainty[12] in the value of their average frequency to allow them to claim rigorous traceability to SI, in keeping with their understanding of the *Guide to the Expression of Uncertainty in Measurement.*

The frequency difference measurements made in the calibration are generally combined linearly, and the expected standard uncertainty at a user interval can be calculated for the general case in a way similar to that outlined for least-squares fitting.[10] The calibration data have a specific structure, and there is a "structure factor" which will independently affect each noise type of the usual sum of power-law noises, with the noise amplitude given by the Allan deviation or

modified Allan deviation. There will be no cross-terms of mixed noise types. The structure factors will also depend on the effective bandwidths for the Allan deviation determination, the calibration interval, and the end-use interval. Boulanger[11,6,10] has developed a convenient analytic form of the required cross-correlations, but the problem has appeared rather too messy for most tastes. Using Boulanger's methods, the "AB structure factors" (the correction factor by which to multiply the Allan deviation to obtain the standard deviation) might be determined for specific calibration types and holdover times. If it is important to distinguish the effects of white phase noise, we would expect to have to use the modified Allan deviation and use a "MAB structure factor."

Recently we have derived, and plan to use a practical method (simple enough for hand calculator use) for converting the Allan variance $\sigma_y^2(\tau)$ measure of stability into a rigorous estimate of the standard uncertainty of average frequency due to the transfer process from one general time interval to another. The method is applicable when the measured $\sigma_y^2(\tau)$ reveals that the stability can be modelled by a sum of phase noise (white and flicker phase noise), white frequency noise, flicker frequency noise, and random-walk frequency noise.

The simple method applies to the commonly used endpoint, or strict average frequency transfer without frequency drift. (Although the standard uncertainty with drift and/or with least-squares fits could be estimated in the same general way, they would still be intricate to use.) The first simplification is to consider what will commonly be possible to arrange: to keep the effective noise bandwidths the same for the Allan deviation characterization, for the calibration interval $[t_1,t_2]$, and for the end-use interval $[t_3,t_4]$. The ratio of the (standard uncertainty)^2 to Allan variance is calculated for each noise type, and the limit is taken as the low-frequency cutoff tends to zero and the high-frequency cutoff tends to infinity.

We had expected to have to discriminate between the white phase and flicker phase noise, and so use the modified Allan deviation. We were pleasantly surprised to find that this is not required in most cases. As long as the accuracy required for an estimate of the standard deviation is in the normal metrological range of 10-20%, and if no endpoint of the calibration and end-use intervals are closer to any other than 10 times the high-frequency bandwidth, then the expressions in Figure 1 may be used with the Allan deviation, and we may avoid the minor difficulties sometimes encountered in determining the modified Allan deviation from a data set with missing values.

In using the four expressions in Figure 1, one need only decompose the Allan deviation graph for the standard (measured against a standard, and by a measurement system, each having at least a 2-3 times lower Allan deviation). The decomposition gives four numbers (the intercept of four lines $\tau^{-1}, \tau^{-1/2}, \tau^0$, and $\tau^{1/2}$ on the log-log graph with the vertical line with τ equal to the calibration interval τ_1), each is squared and each is multiplied by its AB structure factor specified in Figure 1. The four products are summed, and the square root is taken to obtain the standard deviation in the average frequency over the interval τ_2, after a holdover time of t, due to the random instability of the frequency standard. Of course, all other sources of average frequency uncertainty must still be added in quadrature to this frequency-transfer uncertainty to obtain the final answer desired by the client. The method also works rigorously for negative t, corresponding to postprocessing of the calibration data.

As one example of postprocessing, we have considered the effect of different local oscillators for average-frequency transfer in field-calibrating GPS-disciplined oscillators, transferring the common-view or wide-area differential GPS 24-hour frequency average to a shorter, centered interval. The results are graphed, in the common way in metrology as the 2-σ uncertainty, in Figure 2 as the dashed lines. The local oscillators considered are modelled conservatively:

- Cs HP5071A (PM 1.5×10^{-10} at .01 s or 1.7×10^{-17} at 24 h, white FM of 2.3×10^{-13} at 24 h, flicker FM of 2×10^{-14}, and random walk FM of 10^{-16} at 24 h),
- Cs HP5071A (high performance: PM 1.5×10^{-10} at .01 s or 1.7×10^{-17} at 24 h, white FM of 3.8×10^{-14} at 24 h, flicker FM of 0.8×10^{-14}, and random walk FM of 10^{-16} at 24 h),
- a good Rb HP5065A (PM 1.5×10^{-10} at .01 s or 1.7×10^{-17} at 24 h, white FM of 5×10^{-13} at 100 s or 1.7×10^{-14} at 24 h, flicker FM of 1×10^{-13}, and random walk FM of 10^{-13} at 24 h), and
- H maser Kvarz CH1-75 (PM 3×10^{-13} at 1 s or 3.5×10^{-18} at 24 h, white FM of 5.1×10^{-16} at 24 h, flicker FM of 6×10^{-15}, and random walk FM of 10^{-16} at 24 h).

The Cs standards are in their white frequency noise regime, and so their standard uncertainty for frequency transfer from 24 hours to short time intervals could have been estimated easily without the AB structure factors, but only by using the AB structure factors could we form a reasonable estimate for the capabilities of the Rb standard and the H maser, which are dominated by non-white frequency noise at 24 h.

This example is simple enough so that field calibration might be done by an energetic client, using a local oscillator certified for frequency transfer, or by a system comprised of a travelling phase-comparator, a frequency standard, and a GPS receiver used in the common-view mode. It would give to clients the means to calibrate, as a function of end-use time interval and time of day, the standard uncertainty $u_y(\tau_2$, time of day) in the average frequency of a GPS-disciplined oscillator at their own site, accounting for the site-dependent and time-dependent variation due to multipath, cabling, temperature, positioning, the ephemerides, the ionosphere, the troposphere, and SA compensation. The competing route is a shorter-term frequency transfer using a real-time wide-area differential GPS network, such as the Canadian ACS, with one or more nodes being a national metrology laboratory responsible for average frequency.

LOW-LEVEL SERVICES

As with many time and frequency groups, our highest public profile is as the identifiable national experts on time. We provide official time for Canada: disseminating it by telephone, radio networks, and short-wave radio. Radio station CHU broadcasts continuously at 3.33 MHz, 7.335 MHz, and 14.67 MHz. Our only long-term plans concerning CHU note the requirement that by the year 2007, the 7.335 MHz frequency allocation will be reserved for broadcast only.

These services provide English voice announcements, French voice announcements, and Bell 103 decodeable FSK signals at 300 bps, both by telephone and short-wave radio. These services provide traceable time (when the receiver and decoder are calibrated) and time interval or frequency (when the stability of the decoder is known). They are being used ever more heavily as companies undertake the documentation of traceable calibration as part of their quality control (or ISO-9000) procedures. CHU's Bell 103 readable code also provides one of the cheapest ways for network administrators to access time and date data for implementing a Network Time Protocol (NTP) server. We plan to install NTP servers which are generally accessible, and for the general public not using NTP we would prefer to develop methods to deliver time along with the uncertainty (bounded by the loop time of a time request).

TRACEABLE SERVICES

At a higher level of accuracy than outlined above, we assess client calibration laboratories for average frequency as part of the formal Canadian Laboratory Assessment Service (CLAS). For average frequency, as for all other metrological quantities, client laboratories are expected to maintain an independent local standard and both the means and the practice of establishing and documenting proper statistical control and calibrations relative to a national metrology laboratory. At present, our view is that GPS-disciplined oscillators of the present designs cannot serve both roles. Remote calibration is done by common-view TV line-10 measurements in the metropolitan areas of the NRC time laboratory, by common-view LORAN-C in the vicinity of the Great Lakes LORAN-C chain, by common-view GPS across Canada, and by travelling artefact of known frequency and stability. The Canadian Active Control System is not yet used for average frequency calibrations or stability measurements relative to Canada's national standards. The remote calibration service for short-term average frequency calibration, outlined above, is still in the planning stages, but would be a satisfactory means of establishing the statistical control for present-day GPS disciplined oscillators.

For clients wishing to calibrate time and frequency standards for time, or for average frequency, or for stability, NRC offers calibration services in its laboratory relative to its internal standards or to international time scales. With the stability measurement made at NRC comes the possibility of certifiable capability for average frequency transfer from one time interval to another. At present this can only be done economically if the client's standard has a stability (Allan deviation) which can be decomposed into a sum of power-law noise types. The AB structure factors then would give an analytic form for the expected standard uncertainty for average frequency transfer from one time interval to any other.

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AB Structure Factors: *converting Allan variance $\sigma_y^2(\tau_1)$ into (standard uncertainty)^2 in Average Frequency Transfer across gap t, from τ_1 to τ_2*

1). Decompose and multiply the $\sigma_y^2(\tau_1)$ terms

White / Flicker phase noise by $2(\tau_1^{-2}+\tau_2^{-2}) / (3\tau_1^{-2})$

White frequency noise by

$$\left[\, |t+\tau_1| + |t+\tau_2| + \tau_1 + \tau_2 - |t| - |t+\tau_1+\tau_2| \,\right] / \tau_2$$

Flicker Frequency noise by

$$\left[ln\left[\frac{(\tau_1+t+\tau_2)^2}{\tau_1\tau_2}\right]+\frac{2t+\tau_1}{\tau_2}ln\left|\frac{\tau_1+t+\tau_2}{t+\tau_1}\right|+\frac{2t+\tau_2}{\tau_1}ln\left|\frac{\tau_1+t+\tau_2}{t+\tau_2}\right|+\frac{t^2}{\tau_1\tau_2}ln\left|\frac{t\,(\tau_1+t+\tau_2)}{(t+\tau_1)(t+\tau_2)}\right|\right]/[2ln2]$$

Random walk frequency noise by

$$\left[\, |t|^3 - \tau_1^2\tau_2 - \tau_1\tau_2^2 - |t+\tau_1|^3 - |t+\tau_2|^3 + |t+\tau_1+\tau_2|^3 \,\right] / [2\tau_1^2\tau_2]$$

2). Add and take the square root to get the standard uncertainty due to the transfer.

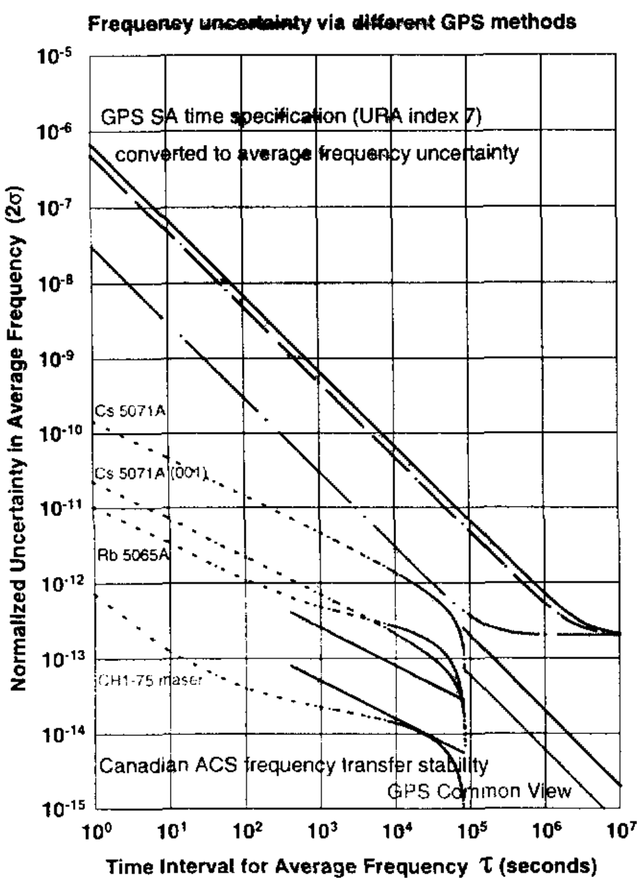

Fig. 1. (above) The simplified method for converting the Allan deviation of an oscillator into its standard uncertainty for the transfer of average frequency from one general time interval to another by decomposition into power law terms and multiplying by the Allan-Boulanger structure factors.

Fig. 2. 2-σ Uncertainty in Average Frequency *vs* Time Interval.
Top: normal SA worst-case correlation
: normal SA uncorrelated
: SA off, medium multipath
: Common view - SA on, daily average
: ACS - SA on, ionosphere measured, carrier phase smoothed, 8 channels.
Dashed curves: standard uncertainty in average frequency from a one-day calibration interval to an end-use interval centered in the calibration interval, of length τ.